Activity 29 Matrix algebra

1.

a) $a=-2,\ b=-1,\ c=-5,\ d=-3$

b) $a=2,\ b=1,\ c=5,\ d=3$

c) $a=1,\ b=-3,\ c=-3,\ d=3$

d) $a=3,\ b=-1,\ c=-5,\ d=2$

e) $a=2,\ b=1,\ c=5,\ d=3$

2.

a)

$$\mathbf{X}=\begin{bmatrix}0&0\\0&0\end{bmatrix}-\begin{bmatrix}2&1\\5&3\end{bmatrix}$$
$$=\begin{bmatrix}-2&-1\\-5&-3\end{bmatrix}$$

b)

$$3\mathbf{X}=\begin{bmatrix}1\\-2\end{bmatrix}\times\begin{bmatrix}18&-5&8\end{bmatrix}-\begin{bmatrix}-3&4&5\\6&-5&2\end{bmatrix}$$
$$\mathbf{X}=\frac{1}{3}\left(\begin{bmatrix}18&-5&8\end{bmatrix}\times\begin{bmatrix}1\\-2\end{bmatrix}-\begin{bmatrix}-3&4&5\\6&-5&2\end{bmatrix}\right)$$
$$=\begin{bmatrix}7&-3&1\\-14&5&-6\end{bmatrix}$$

3.

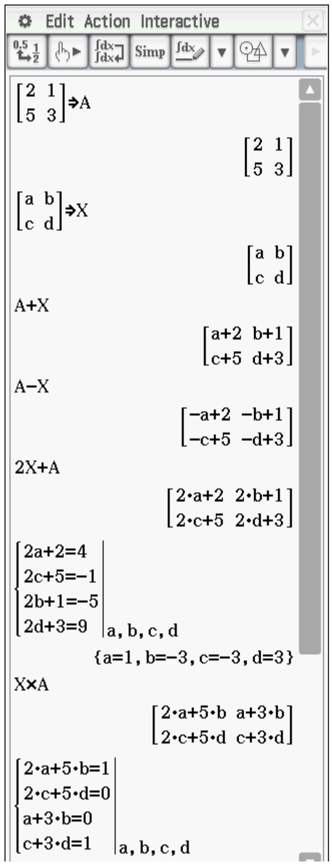
a) $w=\dfrac{d}{ad-bc},x=\dfrac{-b}{ad-bc},y=\dfrac{-c}{ad-bc},z=\dfrac{a}{ad-bc}$

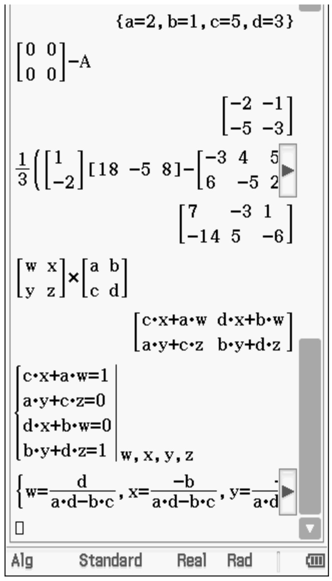
b) $\begin{bmatrix}a&b\\c&d\end{bmatrix}^{-1}=\dfrac{1}{ad-bc}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$

4.

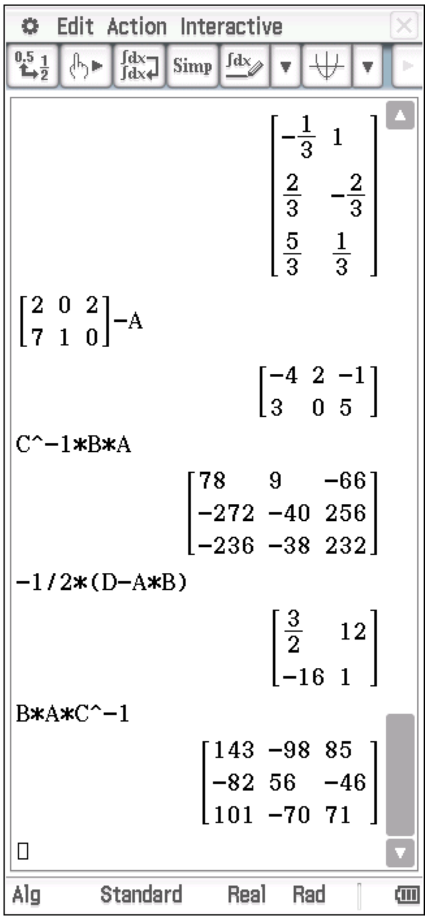
a)

$$\mathbf{X}=\frac{1}{3}\mathbf{B}=\begin{bmatrix}-\frac{1}{3}&1\\\frac{2}{3}&\frac{-2}{3}\\\frac{5}{3}&\frac{1}{3}\end{bmatrix}$$





b) $$\mathbf{X}=\begin{bmatrix}2 & 0 & 2\\ 7 & 1 & 0\end{bmatrix}-\mathbf{A}$$
$$=\begin{bmatrix}-4 & 2 & -1\\ 3 & 0 & 5\end{bmatrix}$$



c) $$\mathbf{CX}=\mathbf{BA}$$
$$\mathbf{C}^{-1}\mathbf{CX}=\mathbf{C}^{-1}\mathbf{BA}$$
$$\mathbf{IX}=\mathbf{C}^{-1}\mathbf{BA}$$
$$\mathbf{X}=\mathbf{C}^{-1}\mathbf{BA}$$
$$=\begin{bmatrix}78 & 9 & -66\\ -272 & -40 & 256\\ -236 & -38 & 232\end{bmatrix}$$

d) $$-2\mathbf{X}=\mathbf{D}-\mathbf{AB}$$
$$\mathbf{X}=-\frac{1}{2}(\mathbf{D}-\mathbf{AB})$$
$$=\begin{bmatrix}\frac{3}{2} & 12\\ -16 & 1\end{bmatrix}$$

e) $$\mathbf{XC}=\mathbf{BA}$$
$$\mathbf{XCC}^{-1}=\mathbf{BAC}^{-1}$$
$$\mathbf{XI}=\mathbf{BAC}^{-1}$$
$$\mathbf{X}=\mathbf{BAC}^{-1}$$
$$=\begin{bmatrix}143 & -98 & 85\\ -82 & 56 & -46\\ 101 & -70 & 71\end{bmatrix}$$

2.

- B[r,c] returns the element in the r^{th} row and c^{th} column of the matrix B.
- dim returns the dimensions of the matrix, i.e. the number of rows and the number of columns.
- det is the determinant. For a 2×2 matrix $\det\begin{bmatrix}a & b\\ c & d\end{bmatrix}=ad-bc$.
- To the power –1 is the inverse of a square matrix, if it exists.
- ident(n) creates an n×n identity matrix. The leading diagonal is 1s; all other elements are 0.
- fill(a,b,c) creates a matrix of a's with b rows and c columns.
- trn(A) swaps rows and columns, i.e. an $m\times n$ matrix becomes an $n\times m$ matrix.