

Activity 29

Matrix algebra

1.

a) $a = -2, b = -1, c = -5, d = -3$

b) $a = 2, b = 1, c = 5, d = 3$

c) $a = 1, b = -3, c = -3, d = 3$

d) $a = 3, b = -1, c = -5, d = 2$

e) $a = 2, b = 1, c = 5, d = 3$

2.

a)

$$\begin{aligned} \mathbf{X} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \\ &= \begin{bmatrix} -2 & -1 \\ -5 & -3 \end{bmatrix} \end{aligned}$$

b)

$$\begin{aligned} 3\mathbf{X} &= \begin{bmatrix} 1 \\ -2 \end{bmatrix} \times [18 \quad -5 \quad 8] - \begin{bmatrix} -3 & 4 & 5 \\ 6 & -5 & 2 \end{bmatrix} \\ \mathbf{X} &= \frac{1}{3} \left(\begin{bmatrix} 1 \\ -2 \end{bmatrix} \times [18 \quad -5 \quad 8] - \begin{bmatrix} -3 & 4 & 5 \\ 6 & -5 & 2 \end{bmatrix} \right) \\ &= \begin{bmatrix} 7 & -3 & 1 \\ -14 & 5 & -6 \end{bmatrix} \end{aligned}$$

3.

a) $w = \frac{d}{ad-bc}, x = \frac{-b}{ad-bc}, y = \frac{-c}{ad-bc}, z = \frac{a}{ad-bc}$

b) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

4.

a)

$$\mathbf{X} = \frac{1}{3} \mathbf{B} = \begin{bmatrix} -\frac{1}{3} & 1 \\ \frac{2}{3} & -\frac{2}{3} \\ \frac{5}{3} & \frac{1}{3} \end{bmatrix}$$

$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \rightarrow \mathbf{A}$
 $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \mathbf{X}$
 $\mathbf{A} + \mathbf{X} = \begin{bmatrix} a+2 & b+1 \\ c+5 & d+3 \end{bmatrix}$
 $\mathbf{A} - \mathbf{X} = \begin{bmatrix} -a+2 & -b+1 \\ -c+5 & -d+3 \end{bmatrix}$
 $2\mathbf{X} + \mathbf{A} = \begin{bmatrix} 2a+2 & 2b+1 \\ 2c+5 & 2d+3 \end{bmatrix}$
 $\mathbf{X} \times \mathbf{A} = \begin{bmatrix} 2a+5b & a+3b \\ 2c+5d & c+3d \end{bmatrix}$
 $\begin{cases} 2a+5b=1 \\ 2c+5d=0 \\ a+3b=0 \\ c+3d=1 \end{cases} \text{ a, b, c, d}$
 $\{a=1, b=-3, c=-3, d=3\}$

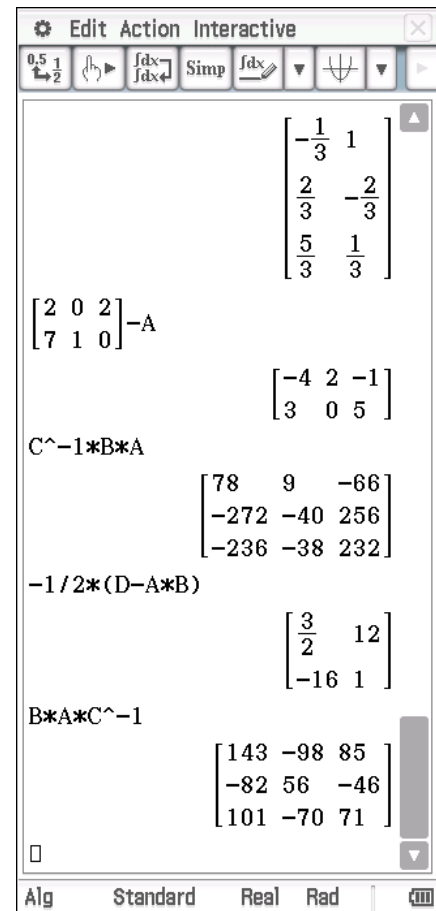
$\{a=2, b=1, c=5, d=3\}$
 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \mathbf{A} = \begin{bmatrix} -2 & -1 \\ -5 & -3 \end{bmatrix}$
 $\frac{1}{3} \left(\begin{bmatrix} 1 \\ -2 \end{bmatrix} \times [18 \quad -5 \quad 8] - \begin{bmatrix} -3 & 4 & 5 \\ 6 & -5 & 2 \end{bmatrix} \right) = \begin{bmatrix} 7 & -3 & 1 \\ -14 & 5 & -6 \end{bmatrix}$
 $\begin{bmatrix} w & x \\ y & z \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c \cdot x + a \cdot w & d \cdot x + b \cdot w \\ a \cdot y + c \cdot z & b \cdot y + d \cdot z \end{bmatrix}$
 $\begin{cases} c \cdot x + a \cdot w = 1 \\ a \cdot y + c \cdot z = 0 \\ d \cdot x + b \cdot w = 0 \\ b \cdot y + d \cdot z = 1 \end{cases} \text{ w, x, y, z}$
 $\left\{ w = \frac{d}{a \cdot d - b \cdot c}, x = \frac{-b}{a \cdot d - b \cdot c}, y = \frac{-c}{a \cdot d - b \cdot c}, z = \frac{a}{a \cdot d - b \cdot c} \right\}$

$$\begin{aligned} \text{b)} \quad \mathbf{X} &= \begin{bmatrix} 2 & 0 & 2 \\ 7 & 1 & 0 \end{bmatrix}^{-1} \mathbf{A} \\ &= \begin{bmatrix} -4 & 2 & -1 \\ 3 & 0 & 5 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{c)} \quad \mathbf{CX} &= \mathbf{BA} \\ \mathbf{C}^{-1}\mathbf{CX} &= \mathbf{C}^{-1}\mathbf{BA} \\ \mathbf{IX} &= \mathbf{C}^{-1}\mathbf{BA} \\ \mathbf{X} &= \mathbf{C}^{-1}\mathbf{BA} \\ &= \begin{bmatrix} 78 & 9 & -66 \\ -272 & -40 & 256 \\ -236 & -38 & 232 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{d)} \quad -2\mathbf{X} &= \mathbf{D} - \mathbf{AB} \\ \mathbf{X} &= -\frac{1}{2}(\mathbf{D} - \mathbf{AB}) \\ &= \begin{bmatrix} \frac{3}{2} & 12 \\ -16 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{e)} \quad \mathbf{XC} &= \mathbf{BA} \\ \mathbf{XC}\mathbf{C}^{-1} &= \mathbf{BAC}^{-1} \\ \mathbf{XI} &= \mathbf{BAC}^{-1} \\ \mathbf{X} &= \mathbf{BAC}^{-1} \\ &= \begin{bmatrix} 143 & -98 & 85 \\ -82 & 56 & -46 \\ 101 & -70 & 71 \end{bmatrix} \end{aligned}$$



2.

- $\mathbf{B}[r,c]$ returns the element in the r^{th} row and c^{th} column of the matrix \mathbf{B} .
- dim returns the dimensions of the matrix, i.e. the number of rows and the number of columns.
- det is the determinant. For a 2×2 matrix $\text{det} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$.
- To the power -1 is the inverse of a square matrix, if it exists.
- $\text{ident}(n)$ creates an $n \times n$ identity matrix. The leading diagonal is 1s; all other elements are 0.
- $\text{fill}(a,b,c)$ creates a matrix of a 's with b rows and c columns.
- $\text{trn}(\mathbf{A})$ swaps rows and columns, i.e. an $m \times n$ matrix becomes an $n \times m$ matrix.